Attitude Stability of Multibody Symmetrical Satellites with Mechanical Dampers

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This paper studies the attitude stability of a satellite consisting of n axisymmetrical bodies spinning at different rates about a common axis, each body containing a nutation damper which moves parallel to the spin axis. The resulting periodic-coefficient differential equations are solved by means of the asymptotic expansion method, thereby permitting a subsequent stability analysis. In the particular case of triple-spin satellites, a numerical parameter analysis based on Floquet theory is carried out to corroborate the analytical stability conditions.

Introduction

THE stabilization of spinning spacecraft by means of energy dissipation is a well-known technique. For a single-body spin-stabilized vehicle, it can easily be shown that the rotational kinetic energy lies within certain limits for a given angular momentum, the minimum energy corresponding to pure spin about the axis of maximum inertia and conversely. This means that a device which can absorb (contribute) kinetic energy when subjected to angular velocities about axes other than the required spin axis will cause the motion to converge towards pure spin about the axis of maximum (minimum) inertia. As passive energy-absorbing devices are the practical choice in most cases, this leads to choosing the axis of maximum inertia as the nominal spin axis. The energy-absorbing devices are called nutation dampers (from the definition of nutation).

For multispin spacecraft, the problem is not so straightforward, and the object of this study is to analyze the stability of axi-symmetrical n-body spacecraft for any inertia distribution and with nutation dampers on any number of the bodies.

Many existing satellites are of the dual- and triple-spin kind, and are therefore particular cases of multispin spacecraft. The general case is considered here in order to cover any advances in the complexity of future satellites, and also to draw some general conclusions regarding the class of spacecraft which may be adequately described by the model we shall study.

The stability of a dual-spin satellite with damping on one of the bodies was solved exactly by Mingori[6] and Likins[7] and the closed form condition they obtained is a particular case of the more general condition found in this paper.

The problem of damped multispin satellites has already been analyzed by Vigneron[5] for spherical rotary dampers, using an averaging method. Our choice of dampers leads to equations of a different nature (periodic coefficient) which lend themselves very well to the application of the asymptotic expansion method. This will reveal the existence of parametric (or internal) resonance cases which are treated separately from the general case.

Satellite Model and Equations of Motion

The satellite model is depicted in Fig. 1. Bodies $M_1 \ldots M_n$ are axisymmetrical and spin at different rates (held constant by means of torque motors) about a common axis of symmetry. The bodies are connected by rigid frictionless bearings, and their spin rates relative to $\{\hat{\mathbf{X}}_\alpha\}$ are constants Ω_i $(i = 1, \ldots, n)$.

Each body contains a damper (mass, spring, and dashpot) which moves parallel to the axis of symmetry: the displacement of the ith damper is noted z_i, its mass m_i and its distance from the spin axis a_i.

$\{\hat{\mathbf{X}}_\alpha\}$ is the system reference frame and is centered at the system center of mass G (defined when all the dampers are at rest). Its angular rate relative to inertial space is $\omega = [\omega_1, \omega_2, \omega_3]^T$.

$\{\hat{\mathbf{x}}_\alpha{}^i\}$ is the ith-body frame and is centered at the corresponding center of mass C_i when the ith damper is at rest; it is related to $\{\hat{\mathbf{X}}_\alpha\}$ by means of a classical rotation matrix:

$$[\hat{\mathbf{x}}_\alpha{}^i] = A_i[\hat{\mathbf{X}}_\alpha], A_i = \begin{bmatrix} \cos\Phi_i & \sin\Phi_i & 0 \\ -\sin\Phi_i & \cos\Phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\Phi_i = \Omega_i t$ is the angle of rotation of $\{\hat{\mathbf{x}}_\alpha{}^i\}$ relative to $\{\hat{\mathbf{X}}_\alpha\}$.

The equations of motion are of two types: rotational equations in their Euler formulation for the variables $\omega_1, \omega_2, \omega_3$, and translational equations in their Lagrange formulation for the displacements $z_1, z_2, \ldots, z_n$.

The derivation of these equations is quite arduous and will not be given here; however, all details may be found in Refs. 2 and 4.

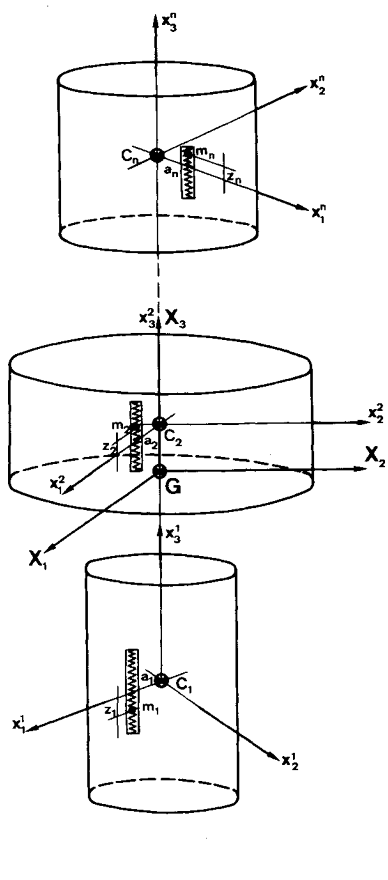

Fig. 1 System model.

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The resulting linearized rotational equations are, for a satellite in field-free space with no external torques

$$\dot{\omega}_1+\omega_0\omega_2+I^{-1}\sum_{i=1}^{n} m_i a_i \sin\Omega_i t[\ddot{z}_i+(\omega_{30}+\Omega_i)^2 z_i]=0$$

$$\dot{\omega}_2-\omega_0\omega_1-I^{-1}\sum_{i=1}^{n} m_i a_i \cos\Omega_i t[\ddot{z}_i+(\omega_{30}+\Omega_i)^2 z_i]=0 \qquad (1)$$

$$\dot{\omega}_3=0$$

where ω_0 is the nutation frequency and I is the total transverse inertia.

This last equation yields $\omega_3=\omega_{30}=$ constant leaving only two rotational variables ω_1 and ω_2 remaining.

As for the displacement equations, their linearized form is

$$m_i(1-\rho_i)\ddot{z}_i-m_i\sum_{\substack{j=1\\ j\neq i}}^{n}\rho_j\ddot{z}_j+m_i a_i\sin\Omega_i t[\dot{\omega}_1+(\omega_{30}+2\Omega_i)\omega_2]-$$
$$m_i a_i\cos\Omega_i t[\dot{\omega}_2-(\omega_{30}+2\Omega_i)\omega_1]+k_i z_i+c_i\dot{z}_i=0$$
$$(i=1,\ldots,n) \qquad (2)$$

where $\rho_i=m_i$/total mass of system, $k_i, c_i=$ spring constant and damping coefficient of ith damper.

Asymptotic Expansion Method

The coefficients of the periodic terms in the Euler equations are of the form $m_i a_i/I$ and may therefore be considered to be of the order of a small parameter ε. Combining the first two equations yields

$$\ddot{\omega}_1+\omega_0^2\omega_1=\varepsilon f_1(y,t) \qquad (3)$$

and the displacement equations may be written

$$\varepsilon(\ddot{y}+J_n y)=\varepsilon f_2(\omega_1,\dot{\omega}_1,\dot{y},t) \qquad (4)$$

where $y=[z_1,\ldots,z_n]^T$; $J_n=n\times n$ matrix whose eigenvalues are equal to the squares of the eigenfrequencies of the dampers when $\omega_1=\omega_2=\omega_3=0$; $f_1=$ analytical scalar function of the displacements and time; and $f_2=$ analytical matrix function of the variables, their time derivatives, and time.

Note that in Eq. (4), both sides are of the same order. This is an important feature of the problem as regards the analytical resolution.

Stability analysis of this system may be carried out in any specific case, by way of the Floquet analysis, but this method does not provide an analytical stability condition.

We will make use of an extension of the asymptotic expansion method due to Bogoliubov and Krylov,[1] and later generalized by Willems and Nishikawa,[3] which is equivalent for linear equations to Floquet analysis and yields a first-order approximation without difficulty. Higher-order approximations may be obtained but require lengthier calculations and have only little practical interest. A full presentation of the first-order approximation theory may be found in Ref. 2. It yields the following results for our problem.

The solution of (3) may be written

$$\omega_1=a\cos\psi_0 \qquad (5)$$

where

$$da/dt=\varepsilon A_0 \qquad (6)$$

$$d\psi_0/dt=\omega_{00}+d\theta_0/dt=\omega_0+\varepsilon B_0 \qquad (7)$$

$$d\theta_0/dt=\varepsilon\eta_0+\varepsilon B_0 \qquad (8)$$

ω_{00} is the value of ω_0 for $\varepsilon=0$. When ε is different from zero, the natural frequency ω_0 will vary slightly; this perturbation will be called "detuning" and we shall write

$$\omega_0=\omega_{00}+\varepsilon\eta_0$$

θ_0 is the variation of ψ_0 around $\omega_{00}t$ for $\varepsilon\neq 0$.

Coefficients A_0 and B_0 are given by the following expressions:

$$A_0=-(1/2\omega_{00})\bar{f}_1^{s*}$$
$$B_0=-(1/2a\omega_{00})\bar{f}_1^{c*} \qquad (9)$$

where $\bar{f}_1^{s*}$ and $\bar{f}_1^{c*}$ are the *total* coefficients of $\sin\psi_0$ and $\cos\psi_0$, respectively, in f_1, obtained by solving Eqs. (4) and introducing the expressions of the z_i into the right-hand side of Eq. (3). We speak of internal resonance when certain linear combinations of periodic terms in the right-hand side of Eq. (3) yield terms in $\sin\psi_0$ and $\cos\psi_0$, thereby augmenting A_0 and B_0. The concept of detuning is of importance only in resonance cases as we shall see further on.

Analytical Resolution and Stability Analysis

Equation (4) is solved by introducing the values of ω_1 and ω_2 for $\varepsilon=0$; i.e., $\omega_1=a\cos\omega_0 t$, $\omega_2=a\sin\omega_0 t$. This yields (to the first order in ε)

$$m_i\ddot{z}_i+c_i\dot{z}_i+k_i z_i=m_i a_i a\{\cos\Omega_i t\cos\omega_0 t[\omega_0-(\omega_{30}+2\Omega_i)]+$$
$$\sin\Omega_i t\sin\omega_0 t[\omega_0-(\omega_{30}+2\Omega_i)]\}$$

or

$$m_i\ddot{z}_i+c_i\dot{z}_i+k_i z_i=m_i a_i a(\omega_0-\omega_{30}-2\Omega_i)\cos(\Omega_i-\omega_0)t$$

This equation may be rewritten as

$$\ddot{z}_i+2\zeta_i\Omega^i\dot{z}_i+\Omega^{i2}z_i=a_i a(\omega_0-\omega_{30}-2\Omega_i)\cos(\omega_0-\Omega_i)t$$
$$=\Omega^{i2}p_i\cos(\omega_0-\Omega_i)t \qquad (10)$$

where

$$\Omega^{i2}=(k_i/m_i)$$
$$\zeta_i=(c_i/2\Omega^i m_i)$$
$$p_i=aa_i(\omega_0-\omega_{30}-2\Omega_i)/\Omega^{i2}$$

The solution of (10) is well known and reads

$$z_i=p_i[(1-r_i^2)^2+4\zeta_i^2 r_i^2]^{-1/2}\cos[(\omega_0-\Omega_i)t-\phi_i],$$

where

$$r_i=(\omega_0-\Omega_i)/\Omega^i,$$
$$tg\phi_i=2\zeta_i r_i/(1-r_i^2).$$

This may also be written

$$z_i=p_i[(1-r_i^2)^2+4\zeta_i^2 r_i^2]^{-1}[(1-r_i^2)\cos(\omega_0-\Omega_i)t+$$
$$2\zeta_i r_i\sin(\omega_0-\Omega_i)t]$$

We now substitute the values of z_i (where $\omega_0 t$ becomes ψ_0 as we now consider $\varepsilon\neq 0$) into Eq. (3) and apply the results [Eqs. (5–8).]

A. Nonresonant Case

In this case we need only determine the coefficients of $\sin\psi_0$ and $\cos\psi_0$ which appear directly in the right-hand side of Eq. (3), without considering any combinations of frequencies which could yield additional terms in $\sin\psi_0$ and $\cos\psi_0$.

This operation yields

$$\frac{I\varepsilon\bar{f}_1^{s*}}{2\omega_0}=-(\omega_0+\omega_{30})\Sigma m_i a_i(\omega_{30}-\omega_0+2\Omega_i)\zeta_i r_i p_i[(1-r_i^2)^2+$$
$$4\zeta_i^2 r_i^2]^{-1}\triangleq Ia\xi$$

thereby defining ξ.

We see that $\bar{f}_1^{s*}$ is a function of a only (and not of the phases) which was to be expected as the system is linear and nonresonant. We will therefore be interested only in $\bar{f}_1^{s*}$ as regards the stability of the system.

From Eqs. (6) and (9), we may write

$$da/dt+a\xi=0$$

where

$$a=a_0e^{-\xi t} \qquad (11)$$

Stability will occur when $\xi>0$ (when $\xi=0$, no conclusion may be drawn as to the stability of the original nonlinear system and a second-order approximation would have to be developed).

Developing ξ yields the condition

$$(\omega_0+\omega_{30})\Sigma c_i(\omega_0-\Omega_i)(\omega_0-\omega_{30}-2\Omega_i)^2a_i^2\times$$
$$\left\{[\Omega^{i2}-(\omega_0-\Omega_i)^2]^2+\frac{c_i^2(\omega_0-\Omega_i)^2}{m_i^2}\right\}^{-1}>0 \qquad (12)$$

This condition takes a simpler form when all the dampers are tuned; i.e., if:

$$\Omega^i=\omega_0-\Omega_i,\quad i=1,\ldots,n$$

We then have

$$(\omega_0+\omega_{30})\Sigma c_i^{-1}(\omega_0-\Omega_i)^{-1}m_i^2a_i^2(\omega_0-\omega_{30}-2\Omega_i)^2>0, \quad (13)$$

as a *first-order approximation stability condition in the nonresonant case.*

This condition is necessary and sufficient for asymptotic stability in the first-order approximation. The form of the condition permits us to decide quite easily on the effect of a variation of a given parameter (mass, damping coefficient, distance of damper from spin axis, etc.).

When it comes to choosing the body on which to place a unique damper, we see that, if $(\omega_0+\omega_{30})>0$, the best location will be on the slowest rotating body; this is a well known result concerning damping for dual-spin satellites.[6,7]

In this particular case (dual-spin spacecraft with one body damped), our condition reads

$$m^2a^2(\omega_0+\omega_{30})(\omega_0-\omega_{30}-2\Omega)^2/c(\omega_0-\Omega)>0$$

or

$$(\omega_0+\omega_{30})(\omega_0-\Omega)>0$$

which is precisely the condition obtained by Likins[7] and Mingori[6] in this specific case ($\Omega=0$ if the platform carries the damper). The minimum stiffness criterion established by Mingori does not appear here as this minimum is of the order of ε.[6]

B. Resonant Case

The full expression of εf_1 is

$$I^{-1}\sum_{i=1}^{n} m_i a_i[(\omega_0-\Omega_i)^2-(\omega_{30}+\Omega_i)^2]b_i\{\omega_0\cos(\psi_0-\phi_i)+\Omega_i\cos[(\psi_0-2\Omega_i t)-\phi_i]\}, \quad (14)$$

from which $\bar{f}_1^{s*}$ was drawn in the nonresonant case.

The b_i are

$$b_i=p_i[(1-r_i^2)+4\zeta_i^2r_i^2]^{-1/2}$$

Resonance situations occur when

$$\psi_0\cong\Omega_i t, \quad (i=1,\ldots,n) \quad (15)$$

Thus, there are n possible internal resonances.

We recall that

$$(da/dt)=(-\varepsilon/2\omega_{00})\bar{f}_1^{s*}$$

$$\frac{d\psi_0}{dt}=\omega_{00}+\frac{d\theta_0}{dt}=\omega_{00}+\varepsilon\eta_0-\frac{\varepsilon}{2a\omega_{00}}\bar{f}_1^{c*}$$

where $\omega_{00}=\Omega_r$ (taking the rth resonance case).

If we develop expression (14) by separating the terms relative to resonance from the others, we obtain

$$I\varepsilon f_1(a_i,\psi_0)=\sum_{\substack{i=1\\ i\neq r}}^{n} m_i a_i[(\Omega_r-\Omega_i)^2-(\omega_{30}+\Omega_i)^2]\times b_i\{\Omega_r\cos(\psi_0-\phi_i)+\Omega_i\cos[(\psi_0-2\Omega_i t)-\phi_i]\}-m_r a_r(\omega_{30}+\Omega_r)^2b_r\Omega_r[\cos(\psi_0-\phi_r)+\cos(\psi_0-2\theta_0+\phi_r)] \quad (16)$$

The argument $\psi_0-2\theta_0+\phi_r$ is a result of:

$$\psi_0-2\Omega_r t-\phi_r=\omega_{00}t+\theta_0-2\Omega_r t-\phi_r=\theta_0-\omega_{00}t-\phi_r=2\theta_0-\psi_0-\phi_r$$

From Eq. (16), we may now evaluate $\varepsilon\bar{f}_1^{s*}$ and $\varepsilon\bar{f}_1^{c*}$

$$I\varepsilon\bar{f}_1^{s*}=\sum_{\substack{i=1\\ i\neq r}}^{n} m_i a_i\Omega_r[(\Omega_r-\Omega_i)^2-(\omega_{30}+\Omega_i)^2]b_i\sin\phi_i-m_r a_r\Omega_r b_r(\omega_{30}+\Omega_r)^2[\sin\phi_r+\sin(2\theta_0-\phi_r)] \quad (17)$$

$$I\varepsilon\bar{f}_1^{c*}=\sum_{i=1}^{n} m_i a_i\Omega_r[(\Omega_r-\Omega_i)^2-(\omega_{30}+\Omega_i)^2]b_i\cos\phi_i-m_r a_r\Omega_r b_r(\omega_{30}+\Omega_r)^2[\cos\phi_r+\cos(2\theta_0-\phi_r)] \quad (18)$$

Replacing b_i, $\sin\phi_i$, and $\cos\phi_i$ by their expressions then yields

$$\varepsilon\bar{f}_1^{s*}=2A'+2C'\sin 2\theta_0-2D'\cos 2\theta_0,$$
$$\varepsilon\bar{f}_1^{c*}=2B'+2C'\cos 2\theta_0+2D'\sin 2\theta_0,$$

where A', B', C', D' are functions of the parameters of the system.

From Eqs. (6), (7), (8), and (9)

$$(da/dt)=a(A+C\sin 2\theta_0-D\cos 2\theta_0)$$
$$(d\theta_0/dt)=\varepsilon\eta_0+B+C\cos 2\theta_0+D\sin 2\theta_0 \quad (19)$$

where

$$A=-\frac{A'}{\Omega_r a},\quad B=-\frac{B'}{\Omega_r a},\quad C=-\frac{C'}{\Omega_r a},\quad D=-\frac{D'}{\Omega_r a}$$

The stability of the system will thus depend on that of the system of differential equations (19). In order to investigate the stability of Eq. (19), we make the following change of variables:

$$u=a\cos\theta_0$$
$$v=a\sin\theta_0 \quad (20)$$

This yields for Eq. (19)

$$du/dt=(A-D)u+(C-B-\varepsilon\eta_0)v$$
$$dv/dt=(C+B+\varepsilon\eta_0)u+(A+D)v \quad (21)$$

The characteristic equation is

$$\lambda^2-2A\lambda+(B+\varepsilon\eta_0)^2+A^2-C^2-D^2=0 \quad (22)$$

Application of Routh-Hurwitz's criterion yields the following necessary and sufficient conditions for asymptotic stability

$$A<0$$
$$(B+\varepsilon\eta_0)^2+A^2-C^2-D^2>0 \quad (23)$$

The first of these conditions is equivalent to condition (12) in the nonresonant case (for $\omega_0=\omega_{00}$) which was to be expected.

The second condition provides us with the limit values of the detuning $\varepsilon\eta_0$. This means that there is a region of instability in the neighborhood of $\omega_{00}=\Omega_r$. This region may or may not contain $\omega_{00}=\Omega_r$.

If $C^2+D^2-A^2\leqq 0$, then $\varepsilon\eta_{0_{\min}}=0$ and the system is stable if $A<0$. If $C^2+D^2-A^2>0$, the detuning must satisfy one of the following conditions to ensure stability:

$$\varepsilon\eta_0\leqq -B-(C^2+D^2-A^2)^{1/2}$$
$$\varepsilon\eta_0\geqq -B+(C^2+D^2-A^2)^{1/2}$$

The region between these two values in the parameter space is the instability region.

Numerical Analysis

A computer program was written to analyze the stability of any specific case, thereby permitting a parametric stability analysis. The model was a three-body satellite.

This study was based on Floquet theory and consisted in checking the norm of the eigenvalues of a unit fundamental matrix after integration over a period. As is well known, when all the eigenvalues have norms less than unity, the system is stable. This technique was used to study both of the cases covered in the theoretical analysis.

A. Nonresonant Case

For a given set of data, two of the damping coefficients were kept constant, the third varying so as to cross the stability boundary. The value of this parameter on the boundary was then calculated by interpolating, and compared to that resulting from Eq. (12) (dampers were tuned).

The agreement between the theoretical Eq. (12) and numerical results was highly satisfactory. As an example, consider the three following results, all the others being of the same order of discrepancy between actual and predicted limit values:

Case 1

$$\omega_{30}=-0.4$$
$$c_1=5$$
$$c_3=4$$

Theoretical stability region: $c_2\geqq 2.634$.
Stability region by Floquet analysis: $c_2\geqq 2.631$.

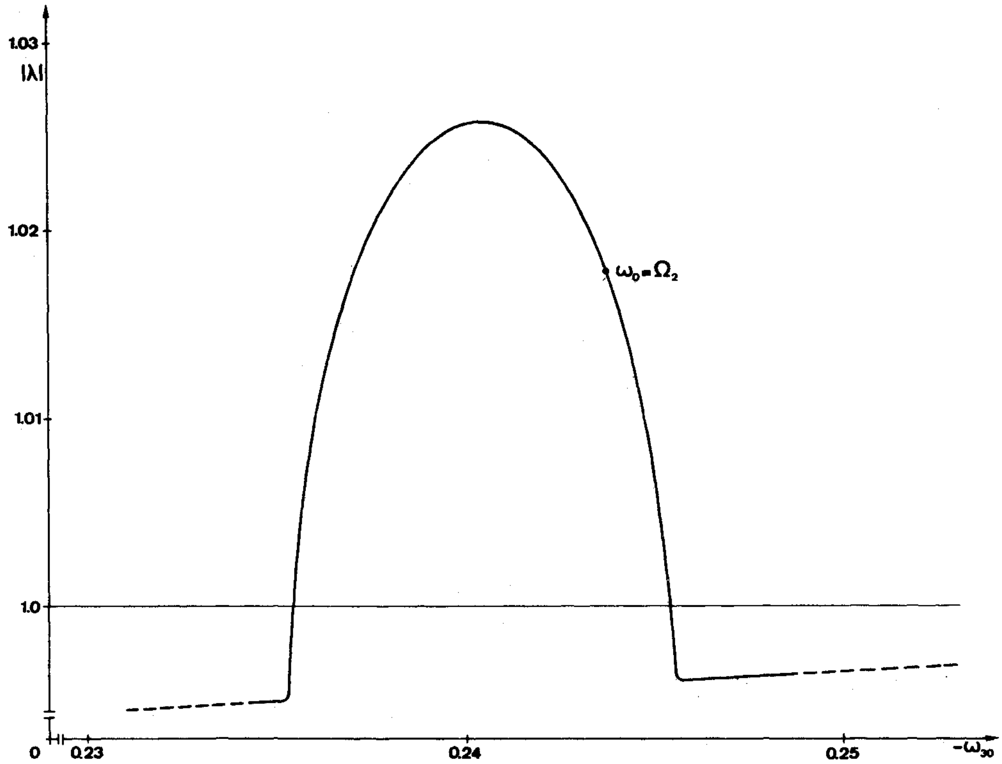

Fig. 2 Variation of $|\lambda|$ in a resonance region for $\Omega_2 = -0.6$.

Case 2

$$\omega_{30} = -0.4$$
$$c_2 = 4$$
$$c_3 = 2$$

Theoretical stability region: $c_1 \leqq 7.928$.
Stability region by Floquet analysis: $c_1 \leqq 7.924$.

Case 3

$$\omega_{30} = -0.6$$
$$c_1 = 5$$
$$c_2 = 0.5$$

Theoretical stability region: $c_3 \leqq 0.0828$.
Stability region by Floquet analysis: $c_3 \leqq 0.0826$.

B. Resonant Case

The damping coefficients were chosen such that stability condition (12) was verified; i.e., the system was stable in the absence of resonance.

The spin rates were chosen such that $\Omega_1 = 0$, $\Omega_3 = n\Omega_2$ (n is an integer) and $\omega_{30,0}$ was then calculated such that $\omega_{00} = \Omega_2$. The parameter ω_{30} was then permitted to vary around $\omega_{30,0}$ (for constant Ω_2 and Ω_3 of course) and the stability was investigated by the same numerical means as for the nonresonant case.

The stability limits were then evaluated by interpolation and, here also, near-perfect agreement between the theoretical Eq. (23) and numerical results was obtained; for example, for $\Omega_2 = -0.6$, $\Omega_3 = -16.2$, the theoretical boundaries of the instability region are (for the chosen data),

$$\varepsilon\eta_{0,1} = -4.794\ 10^{-3}$$
$$\varepsilon\eta_{0,2} = 0.880\ 10^{-3}$$

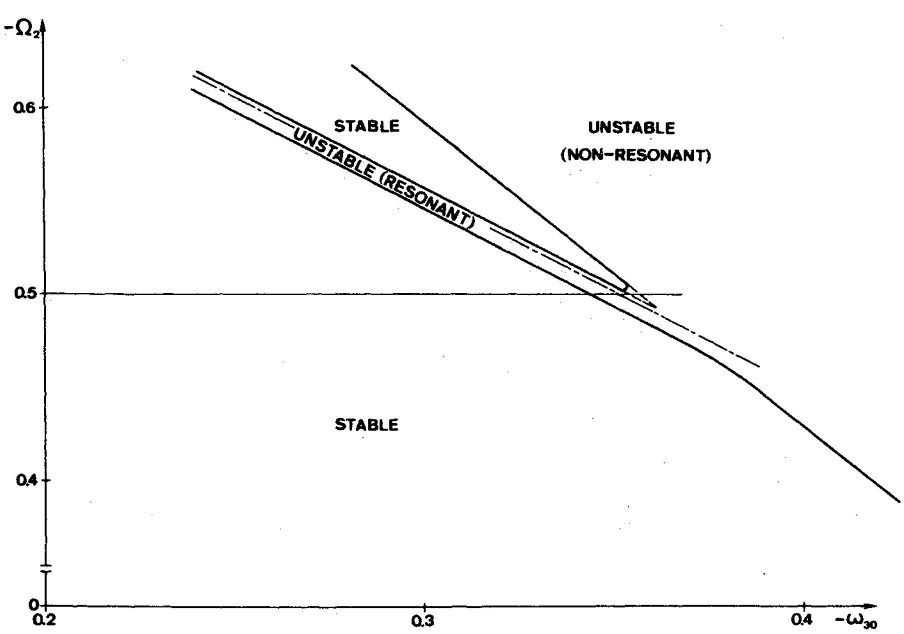

Fig. 3 Stability chart in the (Ω_2, ω_{30}) plane.

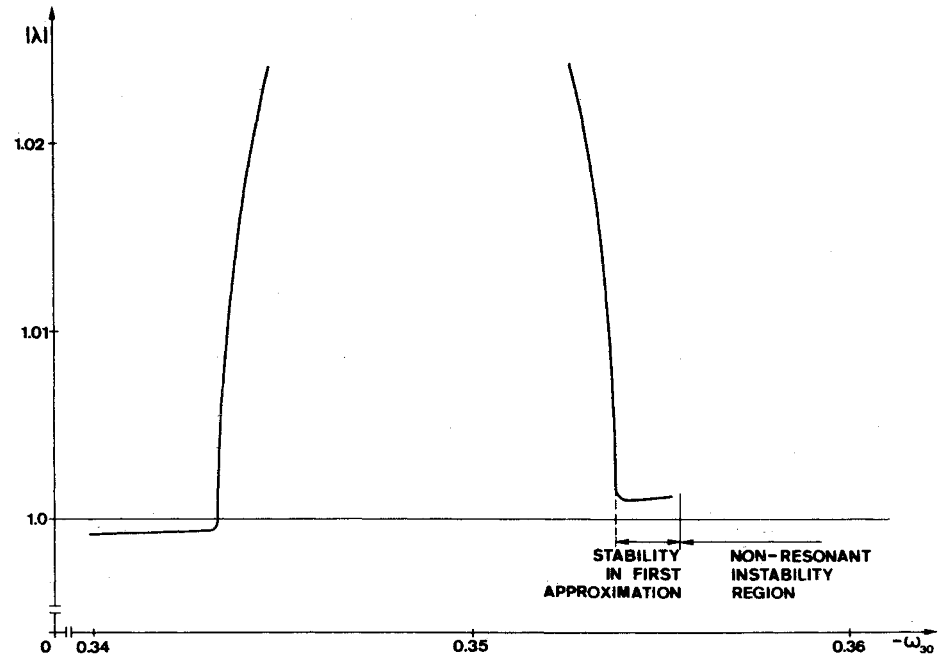

Fig. 4 Variation of $|\lambda|$ in a resonance region for $\Omega_2 = -0.5$.

with $\omega_{00} = \Omega_2 = -0.6$. These values correspond to

$$\omega_{30,0} = -0.243787$$
$$\omega_{30,1} = -0.235169 \qquad (24)$$
$$\omega_{30,2} = -0.245374$$

If we plot $|\lambda|$ as a function of ω_{30} (λ being the Floquet eigenvalue with greatest absolute value) as a result of the numerical investigation, we obtain the variation shown in Fig. 2; we see that $|\lambda|$ undergoes a sudden variation in the vicinity of the resonance instability region and then resumes its slow ascension towards unity [which will be reached at the boundary resulting from Eq. (12)]. We may also note that the slope of the curve on either side of the resonance region is identical and that the curve on the right is a continuation of that on the left: the resonance region may therefore be considered as a perturbation of the normal behavior of the system.

The values of ω_{30} corresponding to $|\lambda| = 1$ are:

$$\omega_{30,1} = -0.2354$$
$$\omega_{30,2} = -0.2454 \qquad (25)$$

and are nearly identical to those predicted theoretically.

If we plot the instability regions in the (Ω_2, ω_{30}) plane (all other parameters remaining constant), the result obtained will be that of Fig. 3 for the set of data considered. At the points where the two instability regions meet, second-order effects cause a deformation of the boundaries. This effect was observed when, for $\Omega_2 = -0.5$, no stability occurred for $\omega_{30} > \omega_{30,0}$, whereas normally, the stable region should have been crossed. In fact, the $|\lambda|$ plot was that of Fig. 4 where we see that $|\lambda|$ departs from its slow ascent as in Fig. 2 and crosses $|\lambda| = 1$ for $\omega_{30,1} = -0.3432$ which is in agreement with the theoretical prediction ($\omega_{30,1} = -0.343108$); however, the decrease of $|\lambda|$ after its maximum is halted suddenly and $|\lambda|$ remains greater than unity, its graph joining up with the slow ascent when already in the nonresonant instability region (if we extrapolate the decreasing portion of the $|\lambda|$ curve, we obtain $\omega_{30,2} = -0.3538$ which may be compared with the theoretical prediction $\omega_{30,2} = -0.353724$).

This strange phenomenon may be explained by considering second-order effects in the vicinity of the boundaries-intersection point, and corresponds to results already obtained by Willems,[8] while studying the attitude stability of a rigid symmetrical spinning satellite on an elliptical orbit (where the small quantity ε is the eccentricity of the orbit).

The limiting values found in both cases corroborated the theoretical results with a relative error of less than 10^{-3} for the chosen data: this error is of the order of ε^2 which was to be expected.

Conclusion

A first-order asymptotic expansion method enabled us to obtain analytical stability conditions with a high degree of accuracy.

The nonresonant stability condition leads to preferring certain bodies when deciding on the location of the damper(s): this choice results from condition (12) and conforms to previous results in specific cases (e.g., dual-spin spacecraft). Consideration of resonant cases extends the instability region in parameter-space, by adjoining to the nonresonant instability region small instability strips in the vicinity of resonance hyperplanes. It should also be noted that at exact resonance, the system can be stable. This occurs when both limit values of the detuning have the same sign, i.e., when $A^2+B^2-C^2-D^2 > 0$.

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